An Illustrated Guide to the
ANALYTIC HIERARCHY PROCESS

Oliver Meixner & Rainer Haas
Institute of Marketing & Innovation
University of Natural Resources and Life Sciences, Vienna

http://www.wiso.boku.ac.at/mi/

Version Aug. 2017

In remembrance & honour of

Thomas Saaty
1926 – 2017

https://www.forevermissed.com/thomas-saaty
Do your decision conferences turn out like this?

WE WANT PROGRAM A!!

TOO BAD! WE WANT PROGRAM B!!

or does this happen?

COME ON IN
THE WATER IS FINE!

sea of indecision

DO YOUR RECOMMENDATIONS TURN OUT LIKE THIS?

GUESS AGAIN

BUT BOSS... THAT WAS MY BEST GUESS!

MAYBE YOU NEED A NEW APPROACH
I THINK I ‘LL TRY THE ANALYTIC HIERARCHY PROCESS (AHP) !!!

... another way of decision making

OKAY TELL US ABOUT AHP

PROF. DR THOMAS L. SAATY DEVELOPED THE PROCESS IN THE EARLY 1970’S AND...
THE PROCESS HAS BEEN USED TO ASSIST NUMEROUS CORPORATE AND GOVERNMENT DECISION MAKERS.

Some examples of decision problems:
- choosing a telecommunication system
- formulating a drug policy
- choosing a product marketing strategy
- ...

Let's show how it works

PROBLEMS ARE DECOMPOSED INTO A HIERARCHY OF CRITERIA AND ALTERNATIVES
OKAY, HERE’S A DECISION PROBLEM WE FACE IN OUR PERSONAL LIVES

I SEE A NEW CAR IN YOUR FUTURE
• STATE THE OBJECTIVE:
  – SELECT A NEW CAR
• DEFINE THE CRITERIA:
  – STYLE, RELIABILITY, FUEL ECONOMY
• PICK THE ALTERNATIVES:
  – CIVIC COUPE, SATURN COUPE, FORD ESCORT, RENAULT CLIO

WHAT ABOUT COST?

(BE QUIET, WE’LL TALK ABOUT THAT LATER)

THIS INFORMATION IS THEN ARRANGED IN A HIERARCHICAL TREE
THE INFORMATION IS THEN SYNTHESIZED TO DETERMINE RELATIVE RANKINGS OF ALTERNATIVES.

BOTH QUALITATIVE AND QUANTITATIVE CRITERIA CAN BE COMPARED USING INFORMED JUDGMENTS TO DERIVE WEIGHTS AND PRIORITIES.

 HOW DO YOU DETERMINE THE RELATIVE IMPORTANCE OF THE CRITERIA?

Here's one way!

STYLE
RELIABILITY
FUEL ECONOMY
Hmm, I think reliability is the most important followed by style and fuel economy is least important so I will make the following judgments ....

**HERE’S ANOTHER WAY**

USING JUDGMENTS TO DETERMINE THE RANKING OF THE CRITERIA

1. RELIABILITY IS 2 TIMES AS IMPORTANT AS STYLE
2. STYLE IS 3 TIMES AS IMPORTANT AS FUEL ECONOMY
3. RELIABILITY IS 4 TIMES AS IMPORTANT AS FUEL ECONOMY

He’s not very consistent here ... that’s o.k.

Pairwise Comparisons

USING PAIRWISE COMPARISONS, THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER CAN BE EXPRESSED
## Pairwise Comparisons

**USING PAIRWISE COMPARISONS, THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER CAN BE EXPRESSED**

<table>
<thead>
<tr>
<th></th>
<th>STYLE</th>
<th>RELIABILITY</th>
<th>FUEL ECONOMY</th>
</tr>
</thead>
<tbody>
<tr>
<td>STYLE</td>
<td>1/1</td>
<td>1/2</td>
<td>3/1</td>
</tr>
<tr>
<td>RELIABILITY</td>
<td>1/1</td>
<td>4/1</td>
<td></td>
</tr>
<tr>
<td>FUEL ECONOMY</td>
<td></td>
<td>1/1</td>
<td></td>
</tr>
</tbody>
</table>

1 equal   3 moderate   5 strong   7 very strong   9 extreme

---

**Pairwise Comparisons**

**USING PAIRWISE COMPARISONS, THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER CAN BE EXPRESSED**

<table>
<thead>
<tr>
<th></th>
<th>STYLE</th>
<th>RELIABILITY</th>
<th>FUEL ECONOMY</th>
</tr>
</thead>
<tbody>
<tr>
<td>STYLE</td>
<td>1/1</td>
<td>1/2</td>
<td>3/1</td>
</tr>
<tr>
<td>RELIABILITY</td>
<td>2/1</td>
<td>1/1</td>
<td>4/1</td>
</tr>
<tr>
<td>FUEL ECONOMY</td>
<td>1/3</td>
<td>1/4</td>
<td>1/1</td>
</tr>
</tbody>
</table>
How do you turn this MATRIX into ranking of criteria?

<table>
<thead>
<tr>
<th></th>
<th>Style</th>
<th>Reliability</th>
<th>Fuel Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Style</td>
<td>1/1</td>
<td>1/2</td>
<td>3/1</td>
</tr>
<tr>
<td>Reliability</td>
<td>2/1</td>
<td>1/1</td>
<td>4/1</td>
</tr>
<tr>
<td>Fuel Economy</td>
<td>1/3</td>
<td>1/4</td>
<td>1/1</td>
</tr>
</tbody>
</table>

How do you get a ranking of priorities from a pairwise matrix?

And the survey says...

EIGENVector!!

Actually...

Prof. Dr. Thomas L. Saaty, (University of Pittsburgh), demonstrated mathematically that the eigenvector solution was the best approach.

HERE’S HOW TO SOLVE FOR THE EIGENVECTOR:

1. A SHORT COMPUTATIONAL WAY TO OBTAIN THIS RANKING IS TO RAISE THE PAIRWISE MATRIX TO POWERS THAT ARE SUCCESSIVELY SQUARED EACH TIME.

2. THE ROW SUMS ARE THEN CALCULATED AND NORMALIZED.

3. THE COMPUTER IS INSTRUCTED TO STOP WHEN THE DIFFERENCE BETWEEN THESE SUMS IN TWO CONSECUTIVE CALCULATIONS IS SMALLER THAN A PRESCRIBED VALUE.

**SAY WHAT!**

**SHOW ME AN EXAMPLE**

---

IT’S MATRIX ALGEBRA TIME !!!

<table>
<thead>
<tr>
<th>STYLE</th>
<th>RELIABILITY</th>
<th>FUEL ECONOMY</th>
</tr>
</thead>
<tbody>
<tr>
<td>STYLE</td>
<td>1/1</td>
<td>1/2</td>
</tr>
<tr>
<td>RELIABILITY</td>
<td>2/1</td>
<td>1/1</td>
</tr>
<tr>
<td>FUEL ECONOMY</td>
<td>1/3</td>
<td>1/4</td>
</tr>
</tbody>
</table>

FOR NOW, LET’S REMOVE THE NAMES AND CONVERT THE FRACTIONS TO DECIMALS :

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0000</td>
<td>0.5000</td>
<td>3.0000</td>
</tr>
<tr>
<td>2.0000</td>
<td>1.0000</td>
<td>4.0000</td>
</tr>
<tr>
<td>0.3333</td>
<td>0.2500</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
STEP 1: SQUARING THE MATRIX

\[
\begin{bmatrix}
1.0000 & 0.5000 & 3.0000 \\
2.0000 & 1.0000 & 4.0000 \\
0.3333 & 0.2500 & 1.0000 \\
\end{bmatrix}
\times
\begin{bmatrix}
1.0000 & 0.5000 & 3.0000 \\
2.0000 & 1.0000 & 4.0000 \\
0.3333 & 0.2500 & 1.0000 \\
\end{bmatrix}
\]

I.E. (1.0000 \times 1.0000) + (0.5000 \times 2.0000) + (3.0000 \times 0.3333) = 3.0000

RESULTS IN THIS

\[
\begin{bmatrix}
3.0000 & 1.7500 & 8.0000 \\
5.3332 & 3.0000 & 14.0000 \\
1.1666 & 0.6667 & 3.0000 \\
\end{bmatrix}
\]

STEP 2: NOW, LET’S COMPUTE OUR FIRST EIGENVECTOR (TO FOUR DECIMAL PLACES)

FIRST, WE SUM THE ROWS

\[
\begin{align*}
3.0000 & + 1.7500 & + 8.0000 & = 12.7500 & 0.3194 \\
5.3332 & + 3.0000 & + 14.0000 & = 22.3332 & 0.5595 \\
1.1666 & + 0.6667 & + 3.0000 & = 4.8333 & 0.1211 \\
\end{align*}
\]

SECOND, WE SUM THE ROW TOTALS

39.9165 1.0000

FINALLY, WE NORMALIZE BY DIVIDING THE ROW SUM BY THE ROW TOTALS (I.E. 12.7500 DIVIDED BY 39.9165 EQUALS 0.3194)

THE RESULT IS OUR EIGENVECTOR (A LATER SLIDE WILL EXPLAIN THE MEANING IN TERMS OF OUR EXAMPLE)

\[
\begin{bmatrix}
0.3194 \\
0.5595 \\
0.1211 \\
\end{bmatrix}
\]
THIS PROCESS MUST BE ITERATED UNTIL THE EIGENVECTOR SOLUTION DOES NOT CHANGE FROM THE PREVIOUS ITERATION (REMEMBER TO FOUR DECIMAL PLACES IN OUR EXAMPLE)

CONTINUING OUR EXAMPLE, AGAIN, STEP 1: WE SQUARE THIS MATRIX

\[
\begin{pmatrix}
3.0000 & 1.7500 & 8.0000 \\
5.3332 & 3.0000 & 14.0000 \\
1.1666 & 0.6667 & 3.0000
\end{pmatrix}
\]

WITH THIS RESULT

\[
\begin{pmatrix}
27.6653 & 15.8330 & 72.4984 \\
48.3311 & 27.6662 & 126.6642 \\
10.5547 & 6.0414 & 27.6653
\end{pmatrix}
\]

AGAIN STEP 2 : COMPUTE THE EIGENVECTOR (TO FOUR DECIMAL PLACES)

\[
\begin{pmatrix}
27.6653 & + & 15.8330 & + & 72.4984 \\
48.3311 & + & 27.6662 & + & 126.6642 \\
10.5547 & + & 6.0414 & + & 27.6653
\end{pmatrix}
= \begin{pmatrix}
115.9967 & 0.3196 \\
202.6615 & 0.5584 \\
44.2614 & 0.1220
\end{pmatrix}
\]

COMPUTE THE DIFFERENCE OF THE PREVIOUS COMPUTED EIGENVECTOR TO THIS ONE:

\[
\begin{pmatrix}
0.3194 & \rightarrow & 0.3196 & = & -0.0002 \\
0.5595 & \rightarrow & 0.5584 & = & 0.0011 \\
0.1211 & \rightarrow & 0.1220 & = & -0.0009
\end{pmatrix}
\]

TO FOUR DECIMAL PLACES THERE’S NOT MUCH DIFFERENCE HOW ABOUT ONE MORE ITERATION?
I SURRENDER!!
DON'T MAKE ME COMPUTE
ANOTHER EIGENVECTOR

OKAY,OKAY
ACTUALLY, ONE MORE
ITERATION WOULD SHOW
NO DIFFERENCE TO FOUR
DECIMAL PLACES

LET'S NOW LOOK AT
THE MEANING OF THE
EIGENVECTOR

Here's our pairwise
matrix with the names

<table>
<thead>
<tr>
<th></th>
<th>STYLE</th>
<th>RELIABILITY</th>
<th>FUEL ECONOMY</th>
</tr>
</thead>
<tbody>
<tr>
<td>STYLE</td>
<td>1/1</td>
<td>1/2</td>
<td>3/1</td>
</tr>
<tr>
<td>RELIABILITY</td>
<td>2/1</td>
<td>1/1</td>
<td>4/1</td>
</tr>
<tr>
<td>FUEL ECONOMY</td>
<td>1/3</td>
<td>1/4</td>
<td>1/1</td>
</tr>
</tbody>
</table>

And the computed eigenvector gives us the relative ranking of our criteria

<table>
<thead>
<tr>
<th></th>
<th>STYLE</th>
<th>RELIABILITY</th>
<th>FUEL ECONOMY</th>
</tr>
</thead>
<tbody>
<tr>
<td>STYLE</td>
<td>0.3196</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RELIABILITY</td>
<td>0.5584</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FUEL ECONOMY</td>
<td>0.1220</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now back to the hierarchical tree...
HERE’S THE TREE WITH THE CRITERIA WEIGHTS

CRITERIA

Select a new car
1.00

Style
.3196

Reliability
.5584

Fuel Economy
.1220

Civic
Saturn
Escort
Clio

Civic
Saturn
Escort
Clio

WHAT ABOUT THE ALTERNATIVES?

OBJECTIVE

SKEPTIC

I’M GLAD YOU ASKED...

IN TERMS OF STYLE, PAIRWISE COMPARISONS DETERMINES THE PREFERENCE OF EACH ALTERNATIVE OVER ANOTHER

STYLE

<table>
<thead>
<tr>
<th></th>
<th>CIVIC</th>
<th>SATURN</th>
<th>ESCORT</th>
<th>CLIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIVIC</td>
<td>1/1</td>
<td>1/4</td>
<td>4/1</td>
<td>1/6</td>
</tr>
<tr>
<td>SATURN</td>
<td>4/1</td>
<td>1/1</td>
<td>4/1</td>
<td>1/4</td>
</tr>
<tr>
<td>ESCORT</td>
<td>1/4</td>
<td>1/4</td>
<td>1/1</td>
<td>1/5</td>
</tr>
<tr>
<td>CLIO</td>
<td>6/1</td>
<td>4/1</td>
<td>5/1</td>
<td>1/1</td>
</tr>
</tbody>
</table>

AND...
IN TERMS OF RELIABILITY, PAIRWISE COMPARISONS DETERMINES THE PREFERENCE OF EACH ALTERNATIVE OVER ANOTHER

**RELIABILITY**

<table>
<thead>
<tr>
<th></th>
<th>CIVIC</th>
<th>SATURN</th>
<th>ESCORT</th>
<th>CLIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIVIC</td>
<td>1/1</td>
<td>2/1</td>
<td>5/1</td>
<td>1/1</td>
</tr>
<tr>
<td>SATURN</td>
<td>1/2</td>
<td>1/1</td>
<td>3/1</td>
<td>2/1</td>
</tr>
<tr>
<td>ESCORT</td>
<td>1/5</td>
<td>1/3</td>
<td>1/1</td>
<td>1/4</td>
</tr>
<tr>
<td>CLIO</td>
<td>1/1</td>
<td>1/2</td>
<td>4/1</td>
<td>1/1</td>
</tr>
</tbody>
</table>

ITS MATRIX ALGEBRA TIME!!

COMPUTING THE EIGENVECTOR DETERMINES THE RELATIVE RANKING OF ALTERNATIVES UNDER EACH CRITERION (ACTUALLY, WE SQUARED THE MATRIX MORE OFTEN TO GET TO THESE RESULTS)

**RANKING**

<table>
<thead>
<tr>
<th>STYLE</th>
<th>RANKING</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIVIC</td>
<td>3</td>
</tr>
<tr>
<td>SATURN</td>
<td>2</td>
</tr>
<tr>
<td>ESCORT</td>
<td>4</td>
</tr>
<tr>
<td>CLIO</td>
<td>1</td>
</tr>
</tbody>
</table>

**RANKING**

<table>
<thead>
<tr>
<th>RELIABILITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIVIC</td>
</tr>
<tr>
<td>SATURN</td>
</tr>
<tr>
<td>ESCORT</td>
</tr>
<tr>
<td>CLIO</td>
</tr>
</tbody>
</table>

WHAT ABOUT FUEL ECONOMY?

ANOTHER GOOD QUESTION...
AS STATED EARLIER, AHP CAN COMBINE BOTH QUALITATIVE AND QUANTITATIVE INFORMATION

FUEL ECONOMY INFORMATION IS OBTAINED FOR EACH ALTERNATIVE:

<table>
<thead>
<tr>
<th></th>
<th>FUEL ECONOMY (MILES/GALLON)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CIVIC</td>
<td>34</td>
<td>34 / 113 = .3009</td>
</tr>
<tr>
<td>SATURN</td>
<td>27</td>
<td>27 / 113 = .2389</td>
</tr>
<tr>
<td>ESCORT</td>
<td>24</td>
<td>24 / 113 = .2124</td>
</tr>
<tr>
<td>CLIO</td>
<td>28</td>
<td>28 / 113 = .2478</td>
</tr>
</tbody>
</table>

113 / 113 = 1.0000

NORMALIZING THE FUEL ECONOMY INFO ALLOWS US TO USE IT WITH OTHER RANKINGS

HERE'S THE TREE WITH ALL THE WEIGHTS

OKAY, NOW WHAT? I THINK WE'RE READY FOR THE ANSWER...
### A Little More Matrix Algebra Gives Us the Solution:

<table>
<thead>
<tr>
<th>Car</th>
<th>Style</th>
<th>Reliability</th>
<th>Fuel Economy</th>
<th>Criteria Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civic</td>
<td>.1159</td>
<td>.3786</td>
<td>.3009</td>
<td>0.3196 Style</td>
</tr>
<tr>
<td>Saturn</td>
<td>.2468</td>
<td>.2902</td>
<td>.2389</td>
<td>0.5584 Reliability</td>
</tr>
<tr>
<td>Escort</td>
<td>.0600</td>
<td>.0742</td>
<td>.2124</td>
<td>0.1220 Fuel Economy</td>
</tr>
<tr>
<td>Clio</td>
<td>.5773</td>
<td>.2571</td>
<td>.2478</td>
<td></td>
</tr>
</tbody>
</table>

I.E. For the Civic: (.1159 * .3196) + (.3786 * .5584) + (.3009 * .1220) = .2851

\[
\begin{array}{c}
\text{Civic} \\
\text{Saturn} \\
\text{Escort} \\
\text{Clio}
\end{array}
\begin{array}{c}
.2851 \\
.2700 \\
.0865 \\
.3583
\end{array}
\]

AND THE WINNER IS!!!

THE CLIÓ IS THE HIGHEST RANKED CAR

### In Summary, the Analytic Hierarchy Process Provides a Logical Framework to Determine the Benefits of Each Alternative

1. Clio .3583
2. Civic .2851
3. Saturn .2700
4. Escort .0865

WHAT ABOUT COSTS?

WELL, I'LL TELL YOU...
ALTHOUGH COSTS COULD HAVE BEEN INCLUDED, IN MANY COMPLEX DECISIONS, COSTS SHOULD BE SET ASIDE UNTIL THE BENEFITS OF THE ALTERNATIVES ARE EVALUATED

OTHERWISE THIS COULD HAPPEN...

YOUR PROGRAM COST TOO MUCH I DON'T CARE ABOUT ITS BENEFITS

DISCUSSING COSTS TOGETHER WITH BENEFITS CAN SOMETIMES BRING FORTH MANY POLITICAL AND EMOTIONAL RESPONSES

WAYS TO HANDLE BENEFITS AND COSTS INCLUDE THE FOLLOWING:

1. GRAPHING BENEFITS AND COSTS OF EACH ALTERNATIVE

   ![Graph](image)

   CHOSE ALTERNATIVE WITH LOWEST COST AND HIGHEST BENEFIT

2. BENEFIT TO COST RATIOS

3. LINEAR PROGRAMMING

4. SEPARATE BENEFIT AND COST HIERARCHICAL TREES AND THEN COMBINE THE RESULTS

IN OUR EXAMPLE...
**LET’S USE BENEFIT TO COST RATIOS**

(AGAIN, WE HAVE QUANTITATIVE INFORMATION HERE)

<table>
<thead>
<tr>
<th></th>
<th>Normalized Cost $</th>
<th>Normalized Costs</th>
<th>Benefit - Cost Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. CLIO</td>
<td>18,000</td>
<td>.3333</td>
<td></td>
</tr>
<tr>
<td>2. CIVIC</td>
<td>12,000</td>
<td>.2222</td>
<td></td>
</tr>
<tr>
<td>3. SATURN</td>
<td>15,000</td>
<td>.2778</td>
<td></td>
</tr>
<tr>
<td>4. ESCORT</td>
<td>9,000</td>
<td>.1667</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>54,000</strong></td>
<td><strong>1.0000</strong></td>
<td></td>
</tr>
</tbody>
</table>

THE CIVIC IS THE WINNER WITH THE HIGHEST BENEFIT TO COST RATIO

(REMEMBER THE BENEFITS WERE DERIVED EARLIER FROM THE AHP)
AHP CAN BE USED FOR VERY COMPLEX DECISIONS

MANY LEVELS OF CRITERIA AND SUBCRITERIA CAN BE INCLUDED

GOAL

HERE ARE SOME EXAMPLES

AHP CAN BE USED FOR A WIDE VARIETY OF APPLICATIONS

STRATEGIC PLANNING
RESOURCE ALLOCATION
SOURCE SELECTION
BUSINESS/PUBLIC POLICY
PROGRAM SELECTION
AND MUCH MUCH MORE...